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GLOBAL STABILITY OF STATIONARY SOLUTIONS OF REACTION-DIFFUSION SYSTEMS

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GLOBAL STABILITY OF STATIONARY SOLUTIONS OF REACTION-DIFFUSION SYSTEMS

Robert A. Gardner*

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ABSTRACT

is given (is identically equal to)

We give a condition which implies that the trivial solution, $U \equiv 0$, of a class of reaction-diffusion systems with homogeneous Dirichlet boundary conditions, is a global attractor for all non-negative solutions. In certain cases, this condition, which relates the diffusion matrix and the domain to a parameter which depends on the nonlinear term, significantly improves similar conditions which can be obtained from energy estimates. Applications are given to equations arising in mathematical ecology.

AMS (MOS) Subject Classifications - 35B30, 35B40, 35K50, 35K55

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SIGNIFICANCE AND EXPLANATION

Reaction-diffusion systems are systems of nonlinear partial differential equations which arise in various aspects of science and engineering, including mathematical ecology. Such equations can be used to describe the evolution of interacting and diffusing species in a bounded domain, together with the usual assumption that there is no movement across the boundary; however, when there is migration across the boundary, other boundary conditions are more appropriate. It is therefore of interest to study the behavior of solutions of such equations with a variety of different boundary conditions; that is, we suppose that the density of each species at the boundary is zero.

The purpose of this paper is to give a condition which implies that each species in the system moves to extinction. Such conditions are easily obtained from standard "energy" estimates and have been observed by a number of authors. We shall give a condition which, for a reasonable class of equations, provides significantly sharper results. In praticular, under certain conditions we obtain the global stability of extinction for any domain and for arbitrary diffusion rates. Applications are given to equations which describe the ecological interactions of predation and competition.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

GLOBAL STABILITY OF STATIONARY SOLUTIONS OF REACTION-DIFFUSION SYSTEMS

Robert A. Gardner*

I.

The purpose of this paper is to give a condition which implies that the trivial solution $U \equiv 0$ of the reaction-diffusion system

$$U_t = D\Delta U + F(U), \quad U(0,x) = U^0(x) \ , \tag{1}$$

$$U\Big|_{\partial\Omega\times\mathbb{R}_+} = 0 \ , \tag{2}$$

is a global attractor for all non-negative solutions of (1), (2). Our condition relates the diffusion matrix, D, and Ω to a third parameter which depends on the nonlinear term. In certain cases, this provides a considerable improvement over the conditions and results obtained from energy estimates by Conway, Hoff and Smoller [3]. These results have applications to systems arising in mathematical ecology.

We assume that $D = \mathrm{diag}(d_1,\dots,d_n)$, where each d_i is a positive constant, $F(U) = (f_1(U),\dots,f_n(U))$, Δ is the m-dimensional Laplacian, and that Ω is a bounded domain in $\mathbb{R}^m$ lying on one side of its boundary, which we assume to be smooth. The interactive term F is assumed to be smooth and to possess bounded invariant sets Σ of the form $\Sigma = \Pi_{i=1}^n [0,a_i]$, for sufficiently large $a_i > 0$; that is, if $U^0(x)$ is continuous and has values in Σ then $U(x,t) \in \Sigma$ for $x \in \Omega$ and all $t \geq 0$ for which the solution U of (1), (2) exists. This will be the case if we impose the condition

$$F(U)\cdot\nu(U) \leq 0 \ , \tag{3a}$$

for all $U \in \partial\Sigma$ and $x \in \Omega$; here, $\nu(U)$ is an outward normal to $\partial\Sigma$ at U. It follows that there then exists a unique, smooth solution of (1), (2) defined

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for all $t \geq 0$ and with values in Σ; see [2] for details. It will also be assumed that f_i has the form

$$f_i(U) = u_i M_i(U) \tag{3b}$$

In the context of mathematical ecology, M_i is the local growth rate of the i^{th} species.

We first give a condition which implies that the plane $\{u_i = 0\}$ is a global attractor for solutions of (1), (2) with data $U^0(x) \in \Sigma$. Let

$$g_i(w) = \sup\{f_i(\xi_1,\ldots,\xi_{i-1},w,\xi_{i+1},\ldots,\xi_n) : 0 \leq \xi_j \leq a_j, i \neq j\}, \tag{4}$$

and define $G_i(w) = \int_0^w g_i(s)ds$. Now let

$$\lambda_i = \sup\{G_i(w)/w^2 : 0 \leq w \leq a_i\} .$$

(Note that λ_i is bounded by an expression which depends on a_i and the Lipschitz constant of g_i, which is finite; see [4, Lemma].) If R is the radius of the smallest ball containing Ω, our condition is that

$$\lambda_i R^2 d_i^{-1} \gamma < 1 , \tag{5}$$

where $\gamma > 0$ depends only on m if $m > 1$; (if $m = 1$ γ also depends on g_i and $R^2 d_i^{-1}$). The proof employs a comparison technique introduced by Conway and Smoller, [4], to estimate u_i from above by the solution w of the scalar equation

$$w_t = d_i \Delta w + g_i(w), \quad w(x,0) = u_i^0(x), \quad w\big|_{\partial\Omega\times\mathbb{R}_+} = 0 . \tag{6}$$

Next, it is shown that condition (5) implies that zero is the unique solution of the steady state equation

$$0 = d_i \Delta w + g_i(w), \quad w\big|_{\partial\Omega} = 0 , \tag{7}$$

associated with (6), and hence, that $w(x,t)$ must decay to zero as t approaches infinity. Thus if (5) holds for $1 \leq i \leq n$, the origin must be a global attractor. It should be noted that if $\lambda_i \leq 0$, (which occurs in a number of

examples), our condition is independent of D and Ω. In Section III, we apply the above result to equations which describe the ecological interactions of predation and competition.

Acknowledgement. The author would like to thank M. Crandall for suggesting that D and Ω be related to λ_i; (the theorem was originally proved with the hypothesis $\lambda_i \leq 0$ instead of (5)).

II.

We shall begin with a discussion of the case $m = 1$ and $\lambda_i \leq 0$. The proof, though trivial, provides a simple geometric interpretation of condition (5) which may help to extend our results to a more general setting. The crucial step is to show that zero is the unique non-negative solution of (7). If w is a smooth non-negative solution on $\Omega = (-\ell, \ell)$ we multiply (7) by w_x and integrate from zero to x to obtain

$$C = d_i w_x^2/2 + G_i(w)$$

where $C = d_i w_x^2(0)/2 + G_i(w(0))$. Hence the solutions of our boundary value problem coincide with the level curves of the expression $d_i z^2/2 + G_i(w)$, where $z = w_x$. When $G_i(w) \leq 0$, the level curves of this expression are as in Figure 1;

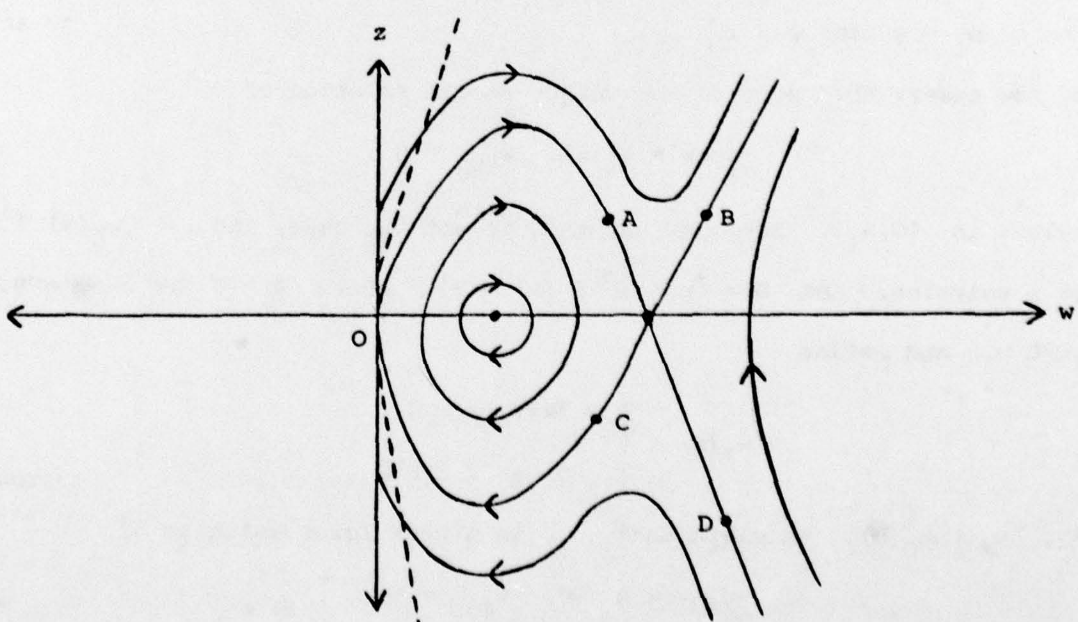

Figure 1.

the solutions of (7) must cross the z-axis when $x = \pm\ell$. Clearly, there is no such smooth non-negative solution. We shall prove a similar result in several space variables by reducing the problem to one dimension.

Theorem. Suppose that conditions (3) and (5) hold. Then $\lim_{t\to\infty} u_i(x,t) = 0$, uniformly for $x \in \Omega$, where u_i is the i^{th} component of the solution of (1), (2).

Proof. We begin by noting that (3) together with the definition (4) of g_i imply that $g_i(0) = 0$ and that $g_i(a_i) \leq 0$, so that 0 and a_i are respectively lower and upper solutions of (7). Moreover, it is shown in [4] that g_i is uniformly Lipschitz (and hence Hölder) continuous on $[0,a_i]$, so that there exists a unique classical solution w of (6) with values in $[0,a_i]$ defined for all $t \geq 0$.

The desired comparison $u_i \leq w$ is obtained by arguing as in [4]. In particular, we let $z = u_i - w$ and $h(x,t,z) = f_i(U) - g_i(w)$, so that z satisfies the equation

$$z_t = d_i \Delta z + h(x,t,z), \quad z\big|_{\partial\Omega\times\mathbb{R}_+} = z\big|_{t=0} = 0 \ . \tag{8}$$

From (4), we see that $h(x,t,0) \leq 0$, so that $\{z \leq 0\}$ is an invariant set for (8); hence $u_i < w$ in $\Omega \times \mathbb{R}_+$.

We now assert that zero is the unique smooth solution of

$$-d_i \Delta w = g_i(w), \quad w\big|_{\partial\Omega} = 0 \ , \tag{9}$$

with values in $[0,a_i]$. Suppose that this is not the case, and let $w_0(x) \not\equiv 0$ be such a solution. Let $D = \{x \in \mathbb{R}^m : |x| < R\}$, where $R > 0$ is chosen such that $\Omega \subset D$, and define

$$w_*(x) = \begin{cases} w_0(x), & x \in \Omega \\ 0 \ , & x \in D\backslash\Omega \ . \end{cases}$$

Clearly, $w_* \in H_0^1(D)$. We claim that w_* is a weak lower solution of

$$-d_i \Delta w = g_i(w), \quad w\big|_{\partial D} = 0 \ ; \tag*{$(9)_D$}$$

that is, w_* satisfies the inequality

$$\int_D d_i \nabla w_* \cdot \nabla \varphi \, dx \leq \int_D g_i(w_*) \varphi \, dx ,$$

for all $\varphi \in H_0^1(D)$ with $\varphi \geq 0$. It clearly suffices to prove this inequality for all $\varphi \in C_0^\infty(D)$. Let n denote the outward unit normal to $\partial\Omega$. Then $dw_0/dn \leq 0$, since $w_0 \geq 0$ in Ω and $w_0 = 0$ on $\partial\Omega$. Let $K = \operatorname{supp}\varphi$, and let $H = \partial\Omega \cap K$. If we give H the orientation induced by n, we have that

$$\int_{\partial(\Omega\cap K)} d_i \varphi \, dw_0/dn \, d\sigma = \int_H d_i \varphi \, dw_0/dn \, d\sigma \leq 0 ,$$

since $\varphi = 0$ on ∂K. Thus

$$\begin{aligned} \int_D d_i \nabla\varphi \cdot \nabla w_* dx &= \int_{K\cap\Omega} d_i \nabla\varphi \cdot \nabla w_* dx + \int_{K\setminus\Omega} d_i \nabla\varphi \cdot \nabla w_* dx \\ &= \int_{\partial(K\cap\Omega)} d_i \varphi \, dw_0/dn \, d\sigma - \int_{K\cap\Omega} d_i \varphi \Delta w_0 dx \\ &\leq \int_{K\cap\Omega} d_i \varphi g_i(w_0) dx = \int_D d_i \varphi g_i(w_*) dx , \end{aligned}$$

since $g_i(0) = 0$. Thus w_* is an $H_0^1(D)$ lower solution of $(9)_D$. Clearly $w^* = a_i$ is an upper solution, so that we may apply an existence theorem of Dueul and Hess, [7], to obtain an exact $H_0^1(D)$ solution w_1 of $(9)_D$ which satisfies $w_* \leq w_1 \leq w^*$. Since w_1 is bounded and g_i is Lipschitz continuous, the usual bootstrapping arguments imply that $w_1 \in C^{2+\alpha}(D)$, so that by an existence theorem of Amann, [1], there exists a maximal $C^{2+\alpha}(D)$ solution w satisfying $w_1 \leq w \leq w^*$. Since any rotation of a solution of $(9)_D$ is again a solution of $(9)_D$, it follows that w is rotationally invariant, so that w satisfies the O.D.E.

$$-d_i(w_{rr} + \frac{m-1}{r} w_r) = g_i(w), \quad w(R) = 0, \quad w_r(0) = 0 ;$$

(the boundary condition at $r = 0$ follows from the smoothness of w). We rescale this equation to obtain

$$-p(w_{rr} + \frac{m-1}{r} w_r) = g_i(w), \quad w(1) = 0, \quad w_r(0) = 0 , \tag{10}$$

where $p = d_i R^{-2}$. Now multiply (10) by w_r and integrate from $r = 0$ to $r = 1$ to obtain

$$p(w_r^2(1)/2 + (m-1)\int_0^1 s^{-1} w_r^2(s)ds) = G_i(w(0)) \quad (11)$$

(We have used the fact that $G_i(w(1)) = G_i(0) = 0$). First, we shall suppose that $m > 1$. Since $w(1) = 0$, by Poincaré's inequality there exists a constant $C_1 > 0$ such that $C_1 \int_0^1 w^2(s)ds \le \int_0^1 w_r^2(s)ds$, so that

$$C_1(1 + C_1)^{-1} \int_0^1 (w^2 + w_r^2)ds \le \int_0^1 w_r^2(s)ds \le \int_0^1 s^{-1} w_r^2(s)ds .$$

By the Sobolev embedding theorem, there exists a constant $C_2 > 0$ such that $C_2\|w\|_\infty \le \|w\|_{H^1}$. The last two inequalities together with (11) imply that $p\gamma^{-1} w(0)^2 \le G_i(w(0))$, where $\gamma^{-1} = C_1C_2^2(m-1)/(C_1+1)$; note that γ depends only on m. Suppose that $w(0) = 0$. Then there exists r_m with $0 < r_m < 1$ and such that $\|w\|_\infty = w(r_m)$. Multiply (10) by w_r and integrate from $r = 0$ to $r = r_m$ to obtain

$$-p(m-1)\int_0^{r_m} s^{-1} w_r^2(s)ds = G_i(w(r_m)) < 0 ,$$

and from $r = r_m$ to $r = 1$ to obtain

$$p(w_r^2(1)/s + (m-1)\int_{r_m}^1 s^{-1} w_r^2(s)ds) = G_i(w(r_m)) > 0 ,$$

contradiction. Thus we have that $w(0) > 0$ and that

$$1 \le p^{-1}\gamma G_i(w(0))w(0)^{-2} \le R^2\gamma\lambda_i/d_i .$$

This inequality violates condition (5), yielding the desired contradiction.

If $m = 1$, we must use a different argument. Let $F(w,z) = (z, -p^{-1}g_i(w))$, so that the solution of (10) also satisfies

$$(w,z)_r = F(w,z), \ (w(1,\beta), z(1,\beta)) = (0,\beta) , \quad (12)$$

when $\beta = \beta_0$ for some $\beta_0 < 0$. Since solutions to (12) are unique, solutions of (12) are uniformly bounded by the particular solution of (12) when $\beta = \beta_0$ for any β with $0 \geq \beta \geq \beta_0$. Viewing β as parameter, we apply Gronwall's inequality to obtain for any β_1, β_2 between zero and β_0

$$(w(0,\beta_1) - w(0,\beta_2))^2 + (z(0,\beta_1) - z(0,\beta_2))^2 \leq \gamma|\beta_1 - \beta_2|^2 ,$$

where γ depends only the Lipschitz constant of $p^{-1}g_i$ and on a_i. Taking $\beta_2 = 0$ and $\beta_1 = \beta_0$, we have that

$$\gamma^{-1}w(0)^2 \leq w_r(1)^2 .$$

Since $m = 1$, we clearly have that $w(0) \neq 0$. The proof now proceeds as above.

We shall now show that $\lim_{t\to\infty} w(x,t) = 0$, pointwise, where $w(x,t)$ is the solution of (6). If this is not the case, there exists $x_0 \in \Omega$ such that $\limsup_{t\to\infty} w(x_0,t) > 0$. By [8, Lemma 3.8], there exists a constant $K > 0$ such that $|\nabla_x w(x,t)| < K$ for $(x,t) \in \Omega \times [1,\infty)$. We may therefore choose a sequence $\{t_n\}$ with $\lim t_n = \infty$, and such that if $w_n(x) = w(x,t_n)$, then $\lim w_n(x) = w_0(x)$ in $C^\alpha(\Omega)$, $0 < \alpha < 1$, and $w_0(x_0) > 0$. We claim that $w_0(x)$ is a solution of (9). If $\varphi \in C_0^\infty(\Omega)$, then

$$\begin{aligned} \left|\int_\Omega [d_i\Delta\varphi w_0 + \varphi g_i(w_0)]dx\right| &= \lim_n \left|\int_\Omega [d_i\Delta\varphi w_n + \varphi g_i(w_n)]dx\right| \\ &= \lim_n \left|\int_\Omega d_i\varphi w_t(x,t_n)dx\right| \\ &\leq \lim_n d_i\|w_t(\cdot,t_n)\|_{L^2}\|\varphi\|_{L^2} . \end{aligned}$$

Now multiply (6) by w_t, integrate over $\Omega \times [1,\infty)$, and apply Green's theorem to obtain

$$\int_1^\infty \int_\Omega w_t^2 dxdt = \lim_{T\to\infty} \left[\int_\Omega (d_i|\nabla w|^2/2 + G_i(w))dx\right]_{t=1}^{t=T} < \infty ,$$

so that $\lim_{t\to\infty}\|w_t(\cdot,t)\|_{L^2} = 0$. Hence w_0 is a weak (and therefore strong)

solution of (9); since g_i is Lipschitz continuous and w_0 is Hölder continuous, we must therefore have that w_0 is a non-trivial, non-negative classical solution, yielding the desired contradiction.

We obtain uniform decay of w as follows. If $\mu(t) = \|w(\cdot,t)\|^2_{L^2}$, we obtain the differential inequality $\mu' + C\mu \leq \int_\Omega wg_i(w)dx$ by multiplying (6) by w, integrating over Ω, integrating by parts, and by finally applying Poincaré's inequality. Since $wg_i(w)$ is a bounded function which converges pointwise to zero, the Lebesgue dominated convergence theorem implies that $\lim_{t\to\infty} \int_\Omega wg_i(w)dx = 0$, and thus the above differential inequality implies that $\lim_{t\to\infty} \mu(t) = 0$. Since w is bounded, we obtain L^p decay for any $p \geq 2$, and we obtain uniform decay from [9, Lemma 3.1]. □

Our result can be extended in several ways. The functions $f_i(U)$ may be allowed to depend on x and t; we need only alter the definition (4) of g_i by taking the supremum of $f_i(U,x,t)$ over all appropriate values of x and t in addition to the variables u_j, $j \neq i$.

We may also replace $-d_i\Delta$ by an elliptic operator L_i of the form

$$L_i = -\sum_{j,k} a^i_{j,k}\partial^2/\partial x_i \partial x_j ,$$

where the $a^i_{j,k}$'s are constants which satisfy the condition $\Sigma_{j,k} a^i_{j,k}\xi_j\xi_k \geq d_i|\xi|^2$. We modify the proof by first performing a linear change of variables $x \to x'$ which transforms L_i into $-d_i\Delta$.

We can also consider other boundary value problems when $m = 1$; (2) is replaced by the mixed condition

$$u_i + \beta_i(x)du_i/dn = 0, \quad x = \pm\ell ,$$

where $\Omega = (-\ell,\ell)$ and $\beta_i \geq 0$. For simplicity, we consider the case $\lambda_i \leq 0$. Suppose that the condition

$$\|\beta_i\|_\infty < \max\{|g_i(w)(-G_i(w))^{-1/2}| : 0 \leq w \leq a_i\} \tag{13}$$

holds. We claim that there are no non-negative, non-trivial stationary solutions. Such a solution must connect the ray $w = \beta_i(-\ell)z$ when $x = -\ell$ to the ray $w = -\beta_i(\ell)z$ when $x = \ell$, where $w_x = z$. Condition (13) ensures that the former ray lies above those trajectories which coincide with the curve OAB in Figure 1 and that the latter ray lies below those trajectories which coincide with the curve OCD in Figure 1, since the slope of OAB and OCD is $\pm g_i(w)(-G_i(w))^{-1/2}$ when $G_i(w) \neq 0$, and the appropriate right or left hand limit of this expression when $G_i(w) = 0$. It should be noted that condition (13) is independent of D and Ω.

The above remarks indicate that our theorem should be true in somewhat greater generality. For example, it is reasonable to consider uniformly elliptic variable coefficient operators of the form $L_i = -\Sigma_{j,k} a^i_{j,k}(x,t)\partial^2/\partial x_i \partial x_j$, or mixed boundary conditions of the form (13) in several space variables. However, the proof given above doesn't seem to be adaptable to such situations. It seems likely that a proof might be found which avoids using the extension procedure and the rotational symmetry of the Laplacian.

III.

In this section, we shall consider a few examples arising in mathematical ecology. Most attention has been given to the Cauchy problem and to the initial-boundary value problem with homogeneous Neumann, (or "no flux") conditions, [4], [5], [6]. However, Dirichlet or mixed boundary conditions may be more appropriate if there is migration across the boundary. It is worth noting that our result is not valid for the pure Neumann or Cauchy problem, since in these cases, there may exist non-trivial constant rest states.

EXAMPLE 1. PREDATION.

Let v be the population density of a predator species, and let u be the density of its prey. Let M and N be the growth rates of u and v respectively; then u and v satisfy the system

$$u_t = d_1 \Delta u + uM(u,v)$$
$$v_t = d_2 \Delta v + vN(u,v) \quad . \tag{14}$$

The interaction is characterized by making certain assumptions about the algebraic signs of M and N and their partial derivatives; for a complete discussion, see [4], [5], [6]. We shall assume that (i), $M \leq 0$ and $N \leq 0$ if u and v are both near zero, (ii), $M \leq 0$ if u is large and $N \leq 0$ if v is large, and (iii), $M_v \leq 0$ and $N_u \geq 0$. Condition (i) requires that there be a critical population density below which each species goes extinct and (ii) is a resource limitation condition. The predator prey relationship is characterized by (iii). One such example is the Rosensweig-MacArthur equations, where we take

$$M = u(-v + \delta(u - a)(b - u))$$
$$N = v(-v + cu - d) \;;$$

$a,b,c,d,$ and δ are positive constants. The phase diagram of the vector field (uM, vN) is given in Figure 2.

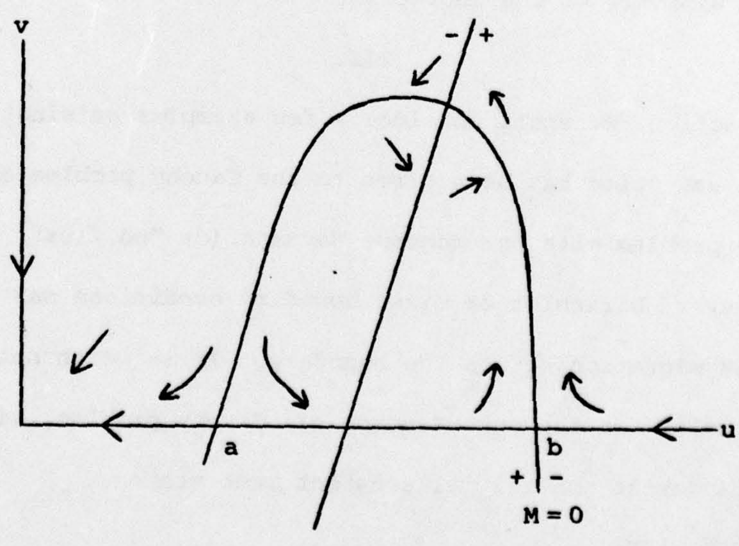

Figure 2.

We have that $g_1(w) = \delta w(w - a)(b - w)$. A simple computation shows that $\lambda_1 = \delta[(a + b)^2/9 - ab/2]$, so that $\lambda_1 \leq 0$ if $a \geq b/2$. In this case, u must tend to zero independently of D, Ω, and δ. Since $N \leq 0$ if u is sufficiently small, we must have that v tends to zero also, (even though λ_2 is always positive here).

EXAMPLE 2. COMPETITION.

We now let u and v be the population densities of two competing species. These variables will again satisfy a system of equations of the form (14). However, we now replace hypothesis (iii) with (iii)', $M_v \leq 0$ and $N_u \leq 0$. (We still assume that (i) and (ii) hold). The phase diagram of an example of such a system is indicated in Figure 3.

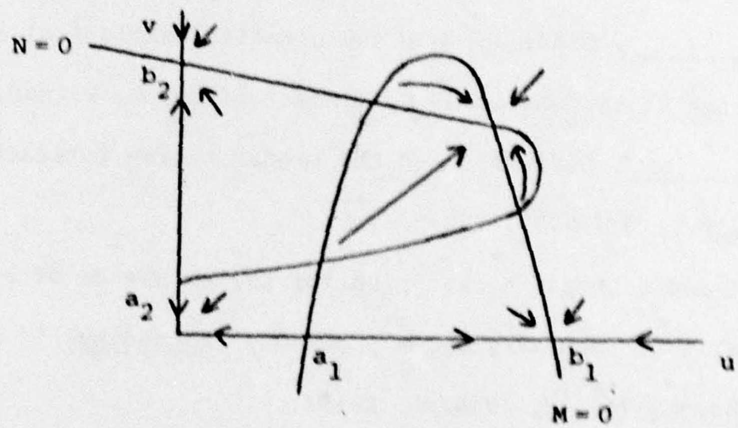

Figure 3.

For definiteness, let

$$M = u(-v + \delta_1(u - a_1)(b_1 - u))$$
$$N = v(-u + \delta_2(v - a_2)(b_2 - v)) .$$

As in Example 1, we have that the origin is a global attractor, provided that $a_i \geq b_i/2$, $i = 1,2$.

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

We give a condition which implies that the trivial solution, $U \equiv 0$, of a class of reaction-diffusion systems with homogeneous Dirichlet boundary conditions, is a global attractor for all non-negative solutions. In certain cases, this condition, which relates the diffusion matrix and the domain to a parameter which depends on the nonlinear term, significantly improves similar conditions which can be obtained from energy estimates. Applications are given to equations arising in mathematical ecology.

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